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Business cycle synchronization according to wavelets – the case of Poland and the euro zone member countries

Joanna Bruzda*

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Abstract

In the paper time-frequency analysis in the form of the maximal overlap discrete wavelet transform (MODWT) and its complex variant – the maximal overlap discrete Hilbert wavelet transform (MODHWT) is applied to study changing patterns of business cycle synchronization between Poland and 8 euro zone member countries (France, Germany, Greece, Ireland, Italy, the Netherlands, Portugal and Spain). We also touch upon the endogeneity hypothesis of the optimum currency area criteria and ask about the recent changes in business cycle variability and their influence on the level of synchronization. Wavelet analysis is a very convenient way of studying business cycles as it possesses good localization properties and is highly efficient in extracting time--varying frequency content of time series. In the paper we make use of these properties and provide a detailed characterization of the degree of business cycle synchronization among the countries under study as well as of the changing amplitudes of business cycles which are measured here as the appropriate frequency components of industrial production indices. In the examination we apply wavelet analysis of variance, wavelet correlation and cross-correlation examination as well as wavelet coherence and wavelet phase angle analysis in their global and (or) local (short-term) versions. The empirical examination points at an increasing synchronization of the Polish business cycle with the euro zone cycles as well as a fairly stable level of business cycle synchronization among the euro zone countries themselves.

Keywords: business cycle synchronization, euro zone, wavelet analysis, maximal overlap discrete wavelet transform, Hilbert wavelet pairs

JEL: C19, E32, E58, O52

* Nicolaus Copernicus University, Department of Logistics; e-mail: bruzdaj@umk.pl.

1. Introduction

There exists a long-standing interest in economics in the causes and mechanisms of business cycles. One of the well-established facts concerning business cycles is their variability over time. This has been stressed already by W.C. Mitchell in his introduction to *Business Annals* (1926, p. 37): 'Recurrence of depression, revival, prosperity and recession, time after time in land after land, may be the chief conclusion drawn from the experience packed into our annals; but a second conclusion is that no two recurrences in all the array seem precisely alike. Business cycles differ in their duration as wholes and in the relative duration of their component phases; they differ in industrial and geographical scope; they differ in intensity; they differ in the features which attain prominence; they differ in the quickness and the uniformity with which they sweep from one country to another.' The last aspect mentioned by Mitchell relates to business cycle synchronization and the phenomenon of an international business cycle.

In the paper time-frequency analysis in the form of the continuous discrete wavelet transform[1] is applied to study changing patterns of business cycle synchronization between Poland and 8 euro zone member countries (France, Germany, Greece, Ireland, Italy, the Netherlands, Portugal and Spain) as well as between the euro zone countries themselves. The study is supplemented with an examination of business cycle variability. The main motivations standing behind the empirical analysis are the following. Firstly, business cycle synchronization is an important factor determining costs of adopting the euro and should be taken into account in the decision-making process concerning entering the euro zone. Secondly, on the experience of the euro zone member countries the endogeneity hypothesis of the optimum currency area (OCA) criteria can be examined, what might constitute another important decision parameter for the candidate countries. Finally, we also ask about the recent changes in business cycle volatility (the end of the Great Moderation) and their influence on the level of synchronization.

Most of the problems mentioned above are not new and are extensively investigated in the literature. Recent studies on business cycle synchronization with(in) the euro zone point at certain diversification among the candidate countries in the degree of synchronization with the euro zone cycle (see, e.g., Fidrmuc, Korhonen 2006; Darvas, Szapáry 2008; Konopczak 2009), a variety in the timing and speed of convergence to the European cycle (Sawa, Neanidis, Osborn 2010), a relatively high degree of synchronization in the case of Poland (see the recent studies of Skrzypczyński 2008; Adamowicz et al. 2009; Konopczak 2009) and usually provide evidence in favour of the endogeneity hypothesis of the optimum currency area criteria (see, e.g., Gonçalves, Rodrigues, Soares 2009, and references therein) with trade intensity being the most important factor standing behind it (see, e.g., de Haan, Inklaar, Jong-A-Pin 2008).

[1] The continuous discrete wavelet transform (CDWT) goes by different names like the non-decimated DWT (NDWT) – see e.g. Nason (2008) – and the maximal overlap DWT (MODWT) – see Percival, Walden (2000). For some other names and their origins see, for example, Mallat (1998), Percival and Walden (2000), who mention also the translation invariant DWT, the stationary DWT and the *algorithme à trous* (algorithm with holes). The acronym CDWT in this context was suggested by Antoniadis and Gijbels (2002) and seems to be a good intuitive reference for the underlying transformation, however we note also that in the wavelet literature it often stands for the complex discrete wavelet transform. Due to this we will mainly use 'maximal overlap DWT' throughout the text, partially also – as Percival and Walden (2000, p. 159), notice – 'because it leads to an acronym that is easy to say ("mod DWT") and carries the connotation of a "modification of the DWT"'.

There is also a growing interest in applying wavelet methodology to examine business cycles and their synchronization and example empirical analyses include Jagrič, Ovin (2004); Raihan, Wen, Zeng (2005); Crowley, Lee (2005); Crowley, Maraun, Mayes (2006); Gallegati, Gallegati (2007); Yogo (2008); Aguiar-Conraria, Soares (2009). Wavelet analysis is a relatively new mathematical concept with a broad range of applications in statistics, image processing and data compression. But wavelets found also their place in modern time series analysis as they make it possible to analyse processes with changing cyclical patterns, trends, structural breaks and other nonstationary characteristics. The distinguishing feature of this technique among other time-frequency methods is an endogenously varying time window, i.e. the ability to analyze short oscillations with narrow time windows (high time resolution) and longer cycles with wider windows (and high frequency resolution). Due to this wavelet methodology is thought of constituting the next logical step in frequency studies, one that elaborates on local time properties of frequency methods. Although the first wavelet was defined in a paper from 1910 (see Haar 1910), wavelet analysis was actually invented in the eighties in France and the United States. The methodology is known to have significant influence on natural sciences (geophysics, oceanography, medicine, etc.), however, with economics and other social sciences it still remains an almost uncharted area with business cycle studies becoming one of the exceptions.

The present study focuses on applications of two continuous discrete wavelet transformations: the MODWT (maximal overlap discrete wavelet transform) and MODHWT (maximal overlap discrete Hilbert wavelet transform). The characteristic feature of the two transformations is that they are continuous in time and discrete in frequency (scales) in the sense that all time units and only octave frequency bands are considered in the analysis. From the point of view of an economist willing to study business cycles the MODWT and MODHWT may offer the following:

– a model-free (nonparametric) approach to examining frequency characteristics of time series, i.e. short, medium and long (as well as other) run features in the series; In particular, due to their nonparametric nature, wavelets enable to examine nonlinear processes without loss of information;

– good time-frequency resolution, and due to this, efficiency in terms of computations needed to extract the features; This enables precise examination of a time-varying frequency content of time series in an efficient way;

– decomposition of variance and covariance of stationary processes according to octave frequency bands (see Percival 1995; Whitcher 1998); In particular, the wavelet variance gives a simplified alternative to the spectral density function, which uses just one value per octave frequency band; The same is true for the wavelet co- and quadrature spectra, which give piecewise constant approximations to the appropriate Fourier cross-spectra on a scale by scale basis;

– precise timing of shocks causing and influencing business cycles;

– low computational complexity; The conventional DWT can be computed with an algorithm that is faster than the well known fast Fourier transform (FFT) – the Mallat's pyramid algorithm, while the computational complexity of the MODWT is exactly the same as the FFT (see Percival, Walden 2000, p. 159);

– examination of trended, seasonal and integrated time series without prior transformations; In particular, we do not need to deseasonalize the data, as seasonal components are left automatically in lower decomposition levels, unless one is interested in examining very short cycles less than two years in length; Besides, there is no need of any prior elimination of deterministic and stochastic trends due to the fact that wavelet filtering usually embeds enough differencing operations;

– efficient estimation of short-term lead-lag relations for octave frequency bands;

– global and local (short-term) measures of association for business cycle components like the wavelet correlations and cross-correlations, wavelet coherences and wavelet phase angles.

Of course, wavelets are not a panacea and typically are as good as other methods (or worse) and their most important characteristics in the present context seem to be an overall computational efficiency and informativeness. These features together with a certain fresh look at an old problem, i.e. operating on octave frequency bands, seem to be the main reasons, why they are worth considering in business cycle examination. In the paper we make use of these properties and provide a detailed characterization of changing patterns of business cycle synchronization between Poland and 8 euro zone countries as well as changing amplitudes of business cycles measured as the appropriate frequency components of industrial production indices. In the examination we apply wavelet analysis of variance, wavelet correlation and cross-correlation analysis as well as wavelet coherence and wavelet phase angle examination in their global and (or) local versions.

The rest of the paper is structured as follows. Sections 2 and 3 describe the types of wavelet transformations that are used further in the empirical examination and discuss issues connected with building confidence intervals for different wavelet quantities. Section 4 presents results of our empirical examination, while the final section shortly concludes.

2. Conventional and maximal overlap discrete wavelet transformations

Wavelet analysis consists in decomposing a signal into shifted and scaled versions of a basic function, $\psi(x)$, called the mother wavelet. The mother wavelet integrates to zero and has unit energy. The discrete wavelet transform (DWT) of a real-valued function $f(x)$ is defined as follows:

$$W_{j,t} = \int_{-\infty}^{\infty} f(x)\psi_{j,t}(x)\,dx \tag{1}$$

where $j = 1, 2, \dots, J$, $t = 0, 1, \dots, 2^{J-j} - 1$ and the wavelet daughters, $\psi_{j,t}(x)$, are shifted and scaled versions of the mother wavelet with dyadic shifts t and scales j, i.e.:

$$\psi_{j,t}(x) = 2^{-j/2}\psi\left(2^{-j}x - t\right) \tag{2}$$

For certain functions $\psi(x)$ with good localization properties $\{\psi_{j,t}(x)\}$ is an orthonormal basis in $L^2(\Re)$. The function $\psi(x)$ is usually defined *via* another function (the scaling function or father wavelet), $\phi(x)$, that applied to the signal after shifting and scaling analogously to (2) produces another set of coefficients in the form:

$$V_{j,t} = \int_{-\infty}^{\infty} f(x)\phi_{j,t}(x)\,dx \tag{3}$$

known as scaling coefficients.

For a given j the wavelet coefficients, $W_{j,t}$, are computed as differences of moving averages for the previous scale scaling coefficients and are associated with scale $\lambda_j = 2^{j-1}$, while their squares contribute to the decomposition of energy of the signal on the time-frequency plane. On the other hand, the level j scaling coefficients are moving averages of scale $\lambda_{j+1} = 2^j$. The two types of coefficients give the multiresolution decomposition of the original function in the form:

$$f(x) = \sum_t V_{J,t}\,\phi_{J,t}(x) + \sum_t W_{J,t}\,\psi_{J,t}(x) + \sum_t W_{J-1,t}\,\psi_{J-1,t}(x) + \cdots + \sum_t W_{1,t}\,\psi_{1,t}(x) = \\ = S_J(x) + D_J(x) + D_{J-1}(x) + \cdots + D_1(x) \tag{4}$$

The functions $S_j(x)$ and $D_j(x)$ are known as approximations (smooths) and details. The highest level approximation $S_J(x)$ represents smooth, low-frequency component of the signal, while the details $D_1(x)$, $D_2(x), \ldots, D_J(x)$ are associated with oscillations of length $2-4, 4-8, \ldots, 2^J - 2^{J+1}$.

In filtering notation the discrete wavelet transform is defined *via* quadrature mirror filters: the low-pass (scaling) filter $\{g_l\}_{l=0,\ldots,L-1}$ and the high-pass (wavelet) filter $\{h_l\}_{l=0,\ldots,L-1}$.[2] The two filters fulfil the quadrature mirror relationship $g_l = (-1)^{l+1} h_{L-1-l}$, have unit energy and are even-shift orthogonal; the wavelet filter integrates (sums) to zero, while the scaling filter – to $\sqrt{2}$. When processing discrete signals we consider a data vector of length $N = 2^J$ in the form $\mathbf{x} = (x_0, x_1, \ldots, x_{N-1})'$. Then the highest possible decomposition level is J and the numbers of wavelet and scaling coefficients of the conventional DWT for each level are $N/2, N/4, \ldots, 1$. On the other hand, the maximal overlap discrete wavelet transform (MODWT) produces the same number of wavelet and scaling coefficients ($\widetilde{W}_{j,t}$ and $\widetilde{V}_{j,t}$, accordingly) at each decomposition level as it does not use downsampling by 2. The coefficients are appropriately scaled in order to retain variance preservation. By definition they are given as follows:

$$W_{j,t} = \sum_{l=0}^{L_j-1} h_{j,l}\, x_{[2^j(t+1)-1-l] \bmod N}, \quad t = 0, \ldots, 2^{J-j} - 1 \tag{5}$$

$$2^{j/2}\widetilde{W}_{j,t} = \sum_{l=0}^{L_j-1} h_{j,l}\, x_{(t-l) \bmod N}, \quad t = 0, \ldots, N-1 \tag{6}$$

$$V_{j,t} = \sum_{l=0}^{L_j-1} g_{j,l}\, x_{[2^j(t+1)-1-l] \bmod N}, \quad t = 0, \ldots, 2^{J-j} - 1 \tag{7}$$

$$2^{j/2}\widetilde{V}_{j,t} = \sum_{l=0}^{L_j-1} g_{j,l}\, x_{(t-l) \bmod N}, \quad t = 0, \ldots, N-1 \tag{8}$$

where $\{h_{j,l}\}$ and $\{g_{j,l}\}$ are the j-th level wavelet and scaling filters of length $L_j = (2^j - 1)(L-1) + 1$ (L is the length of the basic, first stage wavelet filter).[3]

The reconstruction part of wavelet analysis utilizes the inverse wavelet transformation in its conventional or maximal overlap versions, what results in a sequence of details and smooths. Though the details and smooths form an additive decomposition of the signal, the lack of translation invariance of the DWT, on the one hand, and the lack of energy preservation of the MODWT details and smooths, on the other, make them somewhat less attractive in studies concerning business and growth cycle synchronization. They can be helpful, however, in extracting cyclical components of economic time series and, when the MODWT details and smooths are used, in dating business cycle turning points.

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[2] Here we concentrate on compactly supported orthonormal wavelets – see Daubechies (1992), Chapters VI–VIII.

[3] For exact definitions of level j wavelet and scaling filters see Percival, Walden (2000), Chapter 4. For higher decomposition levels j the following approximate relationships hold: $h_{j,l} \approx 2^{-j/2}\psi\left(\frac{l}{2^j}\right)$, $g_{j,l} \approx 2^{-j/2}\phi\left(\frac{l}{2^j}\right)$.

Among the most popular real wavelet filters are the compactly supported orthonormal Daubechies filters: the extremal phase (dL) and the least asymmetric (la$L$) filters (see Daubechies 1992). The two wavelet families are characterized by the smallest filter length L for a given number of vanishing moments (embedded difference operators). Besides, the extremal phase scaling filters have the fastest build-up of the energy sequence, while the least asymmetric filters are approximately linear phase. Figure 1 gives examples of mother and father wavelets corresponding to two of the Daubechies filters. Squared gain functions for filters based on these wavelets are given in Figure 2. They illustrate the fact that by increasing the support length better approximations to ideal bandpass filters are obtained.

The maximal overlap discrete wavelet transform (MODWT) helps to remove certain deficiencies of the conventional discrete wavelet transformation and in our application can be thought of as an improvement over the DWT. Among the distinguishing features of the conventional DWT and the MODWT are the following:

- The MODWT can handle any sample size, while the Jth order partial DWT – only multiplies of 2^J.
- The MODWT is translation invariant, what means that circularly shifting the time series is equivalent to analyzing its circularly shifted wavelet and scaling coefficients or details and smooths.
- As is true for the conventional DWT, the MODWT enables variance and covariance decomposition. But the MODWT provides better estimators of the wavelet variance and covariance (see Whitcher 1998).
- The MODWT approximations and details are associated with zero phase filters, what makes it possible directly to align their features with those from the original series.
- An additive decomposition of the time series in terms of its details and approximations is valid for both the DWT an the MODWT. Contrary to the DWT, however, the MODWT details and approximations do not form an energy (and covariance) decomposition.
- The conventional DWT approximately decorrelates a broad range of stationary as well as nonstationary processes (see Whitcher 1998; Percival, Walden 2000).

Further in the text we concentrate on wavelet analysis of variance, covariance, coherence and phase angle, which are the most important wavelet methods in business cycle studies. To this aim we work with the maximal overlap transformation as it offers better estimators of our wavelet quantities. A more detailed description of the discrete wavelet transform can be found in many mathematical and engineering books (see, e.g., Daubechies 1992; Mallat 1998; Białasiewicz 2004; Nason 2008), while interesting introductions to wavelet analysis from an economic point of view offer Gençay, Selçuk, Whitcher (2002), Ramsey (2002), Schleicher (2002) and Crowley (2007).

For a stochastic process Y_t the time-dependent wavelet variance is defined as:

$$\sigma_t^2(\lambda_j) = \frac{1}{2\lambda_j}\mathrm{Var}(W_{j,t}) = \mathrm{Var}(\widetilde{W}_{j,t}) \tag{9}$$

Assuming that (9) does not depend on time,[4] we arrive at a variance decomposition according to scales λ_j in the form:

[4] The assumption is fulfilled also for nonstationary processes provided that they are integrated of order d and the width of the wavelet filter, L, is sufficient to eliminate nonstationarity. In the case of the Daubechies wavelet filters the condition is $L \geq 2d$. Further, in order to have $E\{\widetilde{W}_{j,t}\} = 0$ we assume that $L > 2d$.

$$\mathrm{Var}(Y_t) = \frac{1}{2}\sum_{j=1}^{\infty}\frac{1}{\lambda_j}\mathrm{Var}(W_{j,t}) = \sum_{j=1}^{\infty}\sigma^2(\lambda_j) \tag{10}$$

The wavelet variance at level j corresponding to scale $\lambda_j = 2^{j-1}$, $\sigma^2(\lambda_j)$, informs about variation of oscillations of length approximately in the interval $2^j - 2^{j+1}$. Similarly, the wavelet covariance and wavelet correlation are introduced. For the stochastic processes X_t and Y_t the time-varying wavelet covariance is defined as follows:

$$\gamma_t(\lambda_j) = \frac{1}{2\lambda_j}\mathrm{Cov}(W^X_{j,t}, W^Y_{j,t}) = \mathrm{Cov}(\widetilde{W}^X_{j,t}, \widetilde{W}^Y_{j,t}) \tag{11}$$

As in the case of the variance decomposition (10), if the wavelet covariances do not depend on time, they produce decomposition of the covariance between X_t and Y_t according to scales λ_j:

$$\mathrm{Cov}(X_t, Y_t) = \frac{1}{2}\sum_{j=1}^{\infty}\frac{1}{\lambda_j}\mathrm{Cov}(W^X_{j,t}, W^Y_{j,t}) = \sum_{j=1}^{\infty}\gamma(\lambda_j) \tag{12}$$

The (time invariant) wavelet correlation coefficient for scale λ_j is defined *via*:

$$\rho(\lambda_j) = \frac{\gamma(\lambda_j)}{\sigma_1(\lambda_j)\,\sigma_2(\lambda_j)} \tag{13}$$

The above quantity measures the strength and direction of linear dependence between two processes for a given decomposition level (scale). Finally, the lag τ wavelet cross-covariance and cross-correlation are given by:

$$\gamma_\tau(\lambda_j) = \frac{1}{2\lambda_j}\mathrm{Cov}(W^X_{j,t}, W^Y_{j,t+\tau}) = \mathrm{Cov}(\widetilde{W}^X_{j,t}, \widetilde{W}^Y_{j,t+\tau}) \tag{14}$$

$$\rho_\tau(\lambda_j) = \frac{\gamma_\tau(\lambda_j)}{\sigma_1(\lambda_j)\,\sigma_2(\lambda_j)} \tag{15}$$

An unbiased and efficient estimator of the wavelet variance is defined as:

$$\hat{\sigma}^2(\lambda_j) = \frac{1}{\widetilde{N}_j}\sum_{t=L_j-1}^{N-1}\widetilde{W}^2_{j,t} \tag{16}$$

where $\widetilde{W}_{j,t}$ are MODWT wavelet coefficients, $L_j = (2^j - 1)(L-1) + 1$ is the length of the wavelet filter for scale λ_j and $\widetilde{N}_j = N - L_j + 1$ is the number of boundary wavelet coefficients (i.e. coefficients unaffected by circularity or reflection at the end of the sample).

An approximate $(1-\alpha)\%$ confidence interval for $\sigma^2(\lambda_j)$ is computed as follows:

$$\hat{\sigma}^2(\lambda_j) \pm \varsigma_{\frac{\alpha}{2}}\left(\frac{\hat{f}_{\overline{W},j}(0)}{\widetilde{N}_j}\right)^{0.5} \tag{17}$$

where $\varsigma_{\frac{\alpha}{2}}$ is the $(1-\alpha/2)$ quantile of the standard normal distribution and $\hat{f}_{\overline{W},j}(0)$ is an estimator of the spectral density of scale λ_j squared wavelet coefficients at frequency 0.

Estimates of wavelet covariances and wavelet correlations are computed *via* the following formulas:

$$\hat{\gamma}(\lambda_j)=\frac{1}{\widetilde{N}_j}\sum_{t=L_j-1}^{N-1}\widetilde{W}_{j,t}^{X}\widetilde{W}_{j,t}^{Y} \tag{18}$$

$$\hat{\rho}(\lambda_j)=\frac{\widetilde{\gamma}(\lambda_j)}{\widetilde{\sigma}_1(\lambda_j)\widetilde{\sigma}_2(\lambda_j)} \tag{19}$$

An approximate $(1-\alpha)\%$ confidence interval fore $\gamma(\lambda_j)$ is obtained as previously with $\hat{f}_{\overline{W},j}(0)$ being an estimate of the spectral density function for the product of scale λ_j wavelet coefficients at frequency 0. To construct confidence intervals for the wavelet correlation the following function must be considered (see Whitcher 1998):

$$g(\bar{a}_j,\bar{b}_j,\bar{c}_j)=\frac{\bar{c}_j}{\sqrt{\bar{a}_j\bar{b}_j}}=\hat{\rho}(\lambda_j) \tag{20}$$

where the mean values $\bar{a}_j,\bar{b}_j,\bar{c}_j$ are defined as:

$$\bar{a}_j=\hat{\sigma}_X^2(\lambda_j)=\frac{1}{\widetilde{N}_j}\sum_{t=L_j-1}^{N-1}(\widetilde{W}_{j,t}^{X})^2$$

$$\bar{b}_j=\hat{\sigma}_Y^2(\lambda_j)=\frac{1}{\widetilde{N}_j}\sum_{t=L_j-1}^{N-1}(\widetilde{W}_{j,t}^{Y})^2$$

$$\bar{c}_j=\hat{\gamma}_{XY}(\lambda_j)=\frac{1}{\widetilde{N}_j}\sum_{t=L_j-1}^{N-1}\widetilde{W}_{j,t}^{X}\widetilde{W}_{j,t}^{Y}$$

Then the vector $[\bar{a}_j,\bar{b}_j,\bar{c}_j]$ has large sample variance given by $\widetilde{N}_j^{-1}S_{abc,j}(0)$, where $S_{abc,j}(\cdot)$ is the 3×3 spectral matrix for $[(\widetilde{W}_{j,t}^{X})^2,(\widetilde{W}_{j,t}^{Y})^2,\widetilde{W}_{j,t}^{X}\widetilde{W}_{j,t}^{Y}]$. By the δ technique[5] the large sample variance of $\hat{\rho}(\lambda_j)$ is $R_{abc,j}(0)/\widetilde{N}_j$, where:

$$R_{abc,j}(0)=\nabla g\left(\sigma_X^2(\lambda_j),\sigma_Y^2(\lambda_j),\gamma_{XY}(\lambda_j)\right)^T\cdot S_{abc,j}(0)\cdot\nabla g\left(\sigma_X^2(\lambda_j),\sigma_Y^2(\lambda_j),\gamma_{XY}(\lambda_j)\right) \tag{21}$$

and ∇g is the gradient of g. Then, an approximate large sample confidence interval for the wavelet correlation is:

$$\hat{\rho}(\lambda_j)\pm\varsigma_{\frac{\alpha}{2}}\left(\frac{\hat{R}_{abc,j}(0)}{\widetilde{N}_j}\right)^{0.5} \tag{22}$$

Other approach takes advantage of the fact that the DWT approximately decorrelates processes and uses the Fisher z-transformation in order to produce intervals bounded by ±1. Under the Gaussian assumption this leads to the following $(1-\alpha)\%$ confidence interval for scale λ_j wavelet coefficients:

[5] The delta technique here works as a generalized central limit theorem which uses Taylor series approximations to nonlinear functions of estimators.

$$\tanh\left\{\tanh^{-1}\left[\hat{\rho}(\lambda_j)\right] \pm \varsigma_{\frac{\alpha}{2}}\left(\frac{1}{\hat{N}_j - 3}\right)^{0.5}\right\} \tag{23}$$

where $\hat{N}_j = N/2^j - L'_j$ is the number of scale λ_j DWT coefficients unaffected by the boundary, which is treated here as a measure of the scale-dependent sample size, and L'_j is the number of boundary DWT coefficients ($L'_j = (L-2)(1-2^{-j})$).

An unbiased estimator of the wavelet cross-covariance is defined as:

$$\hat{\gamma}_\tau(\lambda_j) = \begin{cases} \frac{1}{\tilde{N}_j - \tau} \sum_{t=L_j-1}^{N-\tau-1} \tilde{W}^X_{j,t}\, \tilde{W}^Y_{j,t+\tau} & \text{for } \tau = 0, 1, \ldots, \tilde{N}_j - 1 \\ \frac{1}{\tilde{N}_j - \tau} \sum_{t=L_j-1-\tau}^{N-1} \tilde{W}^X_{j,t}\, \tilde{W}^Y_{j,t+\tau} & \text{for } \tau = -1, -2, \ldots, -(\tilde{N}_j - 1) \\ 0 & \text{otherwise} \end{cases} \tag{24}$$

and the corresponding estimate of the wavelet cross-correlation *via*:

$$\hat{\rho}_\tau(\lambda_j) = \frac{\hat{\gamma}_\tau(\lambda_j)}{\hat{\sigma}_1(\lambda_j)\hat{\sigma}_2(\lambda_j)} \tag{25}$$

Decimation by 2 affects the lag resolution of DWT-based estimators of the wavelet cross--covariances and cross-correlations and they should not be used in practice (see Gençay, Selçuk, Whitcher 2002, p. 252). Confidence intervals for these quantities are obtained by applying similar arguments to presented above. More information about constructing confidence intervals in the wavelet analysis of variance and covariance can be found in Percival (1995), Whitcher (1998), Percival, Walden (2000), Gençay, Selçuk, Whitcher (2002). In the next section we turn to a complex variant of the MODWT called the maximal overlap discrete Hilbert wavelet transform (MODHWT) which makes it possible to perform the wavelet analysis of coherence and phase angle.

3. Maximal overlap discrete Hilbert wavelet transformation

The maximal overlap discrete Hilbert wavelet transform, advocated by Whitcher and Craigmile (2004), makes use of a recently introduced class of filters based on the Hilbert wavelet pairs (HWP) and utilizes the non-decimated (maximal overlap) version of the dual-tree complex wavelet transformation of Kingsbury (2001).[6] The filters in a Hilbert wavelet pair are approximate Hilbert transforms of each other and, as in usual discrete wavelet transformation, form a basis for a collection of orthogonal bandpass filters. This time, however, the approximate analyticity of the filters enables to compute quantities with direct analogy to the Fourier analysis like the wavelet coherence and the wavelet phase angle.

Let $\{h_l^0\}$ and $\{g_l^0\}$ be conjugate quadrature mirror filters, i.e.

[6] For an introduction to the Kingsbury's transform see Selesnick, Baraniuk, Kingsbury (2005).

$$\sum_l h_l^0 = 0; \quad \sum_l (h_l^0)^2 = 1; \quad \sum_l h_l^0 h_{l+2n}^0 = 0; \quad n \neq 0; \quad g_l^0 = (-1)^{l+1} h_{L-1-l}^0$$

and $\{h_l^1\}$ and $\{g_l^1\}$ be another couple of such filters chosen in such a way that their wavelet function is an approximate Hilbert transform of the mother wavelet for the first couple of filters. Figure 3 presents examples of such Hilbert wavelet pairs. They can be seen as localized versions of cosine and sine waves forming the Fourier transformation. As the Hilbert transform is built into a couple of orthogonal wavelet filters, it automatically adapts to the wavelet scales. This feature makes the approach particularly attractive as compared to other time-frequency methods producing instantaneous amplitudes, phases and frequencies like the classic demodulation method (see, e.g., Priestley 1981, §11.2.2) or the modern empirical mode decomposition (see Huang, Shen 2005).

The maximal overlap Hilbert wavelet transform consists in simultaneously applying a pair of wavelet (and scaling) filters in their non-decimated (maximal overlap) forms. As a result, two sequences of coefficients are obtained, which are the real and imaginary parts of the final wavelet coefficients. In other words, the following filters are used:

$$\begin{aligned} \widetilde{h}_{j,l} &= \widetilde{h}_{j,l}^0 + i\widetilde{h}_{j,l}^1 \\ \widetilde{g}_{j,l} &= \widetilde{g}_{j,l}^0 + i\widetilde{g}_{j,l}^1 \end{aligned} \tag{26}$$

These filters produce the complex wavelet and scaling coefficients in the form:

$$\widetilde{W}_{j,t} = \sum_{l=0}^{L-1} \widetilde{h}_{l,j} X_{t-j} = \widetilde{W}_{j,t}^0 + i\,\widetilde{W}_{j,t}^1 \tag{27a}$$

$$\widetilde{V}_{j,t} = \sum_{l=0}^{L-1} \widetilde{g}_{l,j} X_{t-j} = \widetilde{V}_{j,t}^0 + i\,\widetilde{V}_{j,t}^1 \tag{27b}$$

In the empirical part of the paper we utilize the HWP filters introduced by Selesnick (2002). The real and imaginary parts of the Selesnick's filters are of the same length and have the same squared gain function. Under a specified degree of approximation (L) to a Hilbert pair (approximation to analyticity) the Selesnick's procedure produces short filters with a given number of vanishing moments (K). The length of each HWP(K, L) filter is equal $2(K + L)$. In our study we apply mid-phase solutions for HWP(K, L), and denote them *kK*l*L*.

Let $\widetilde{W}_{j,t}^X$ and $\widetilde{W}_{j,t}^Y$ be complex-valued wavelet coefficients obtained through filtering X_t and Y_t. Assuming that the wavelet filters have enough vanishing moments to eliminate deterministic trend components of the series, the time-varying wavelet spectrum of (X_t, Y_t) for scale λ_j is defined as (see Whitcher, Craigmile 2004):

$$\begin{aligned} S_{XY}(\lambda_j, t) &= E\left(\widetilde{W}_{j,t}^X\, \overline{\widetilde{W}_{j,t}^Y}\right) = E\left[\left(\widetilde{W}_{j,t}^{X0} + i\widetilde{W}_{j,t}^{X1}\right)\left(\widetilde{W}_{j,t}^{Y0} - i\widetilde{W}_{j,t}^{Y1}\right)\right] = \\ &= E\left[\left(\widetilde{W}_{j,t}^{X0}\widetilde{W}_{j,t}^{Y0} + \widetilde{W}_{j,t}^{X1}\widetilde{W}_{j,t}^{Y1}\right) - i\left(\widetilde{W}_{j,t}^{X0}\widetilde{W}_{j,t}^{Y1} - \widetilde{W}_{j,t}^{X1}\widetilde{W}_{j,t}^{Y0}\right)\right] = C_{XY}(\lambda_j, t) - i\,Q_{XY}(\lambda_j, t) \end{aligned} \tag{28}$$

where $C_{XY}(\lambda_j, t)$ and $Q_{XY}(\lambda_j, t)$ are the time-varying wavelet cospectrum and quadrature spectrum (quad-spectrum), respectively. If the wavelet cospectrum and quadrature spectrum do not depend on time, it is possible to relate them directly to the appropriate Fourier quantities. We consider two integrated processes: $X_t \sim I(d_X), Y_t \sim I(d_Y)$, whose differences of order d_x and d_y, respectively, are jointly stationary with absolute summable cross-covariance sequences. Then, following Whitcher and Craigmile (2004), the cross-spectral density function is defined as:

$$S_{XY}(f) = \frac{S_{WZ}(f)}{(1-e^{-i2\pi f})^{d_X}\,(1-e^{-i2\pi f})^{d_Y}} \tag{29}$$

where $W_t = \Delta^{d_X} X_t$ and $Z_t = \Delta^{d_Y} Y_t$.

Then, assuming that the wavelet filters have enough vanishing moments, we arrive at the following piecewise constant approximation to the Fourier cross-spectrum:[7]

$$S_{XY}(f) \approx \lambda_j S_{XY}(\lambda_j) \ \text{ for } \ f \in [\tfrac{1}{2^{j+1}}, \tfrac{1}{2^j}] \tag{30}$$

Further, as in Whitcher and Craigmile (2004), we define also the time-varying wavelet cross--amplitude spectrum:

$$A_{XY}(\lambda_j, t) = |S_{XY}(\lambda_j, t)| = \left[C^2_{XY}(\lambda_j, t) + Q^2_{XY}(\lambda_j, t)\right]^{1/2} \tag{31}$$

the time-varying wavelet phase angle:

$$\theta_{XY}(\lambda_j, t) = \operatorname{atan}\left[\frac{-Q_{XY}(\lambda_j, t)}{C_{XY}(\lambda_j, t)}\right] \tag{32}$$

the time-varying wavelet coherence:

$$K_{XY}(\lambda_j, t) = \frac{A^2_{XY}(\lambda_j, t)}{S_X(\lambda_j, t)\,S_Y(\lambda_j, t)} = \frac{C^2_{XY}(\lambda_j, t) + Q^2_{XY}(\lambda_j, t)}{S_X(\lambda_j, t) S_Y(\lambda_j, t)} \tag{33}$$

where $S_X(\lambda_j, t) = E\left|\widetilde{W}^X_{j,t}\right|^2$, $S_Y(\lambda_j, t) = E\left|\widetilde{W}^Y_{j,t}\right|^2$ denote the time-varying wavelet spectra equal 2 times the wavelet variance, $\sigma^2_t(\lambda_j)$.

Besides, we also introduce the time-varying wavelet time delay:

$$\tau_{XY}(\lambda_j, t) \overset{\text{df}}{=} -\frac{\theta_{XY}(\lambda_j, t)}{2\pi f_{j0}} \tag{34}$$

where f_{j0} is the centre frequency of the octave band computed as the arithmetic mean of the upper and lower cutoff frequencies, i.e. $f_{j0} = 3/2^{J+2}$.

[7] See (Bruzda 2011). It is worth mentioning that in order to have good analytic properties of the Hilbert wavelet transform at the first several stages of the Mallat's pyramid algorithm we need a different couple of wavelet filters at the first stage (see Selesnick, Baraniuk, Kingsbury 2005, as well as Bruzda 2011). The level 1 filters are obtained by shifting by one sample basically any real wavelet filter and using it as the second filter in the couple.

If (31)–(34) are constant in time, they may be thought of as average values of the corresponding Fourier amplitude, phase and coherence spectra as well as the frequency dependent time delay in intervals of the form $[1/2^{j+1}, 1/2^{j}]$. Their estimators are obtained by replacing the wavelet cospectrum, quad-spectrum and individual spectra with their estimates computed *via* smoothing in time. The smoothing is necessary, in particular in estimation of the wavelet coherence. A simple two-sided moving average gives estimators in the form:

$$\hat{S}_X(\lambda_j) = \frac{1}{\widetilde{N}_j} \sum_{t=L_j-1}^{N-1} \left| \widetilde{W}_{j,t}^X \right|^2 \tag{35a}$$

$$\hat{S}_Y(\lambda_j) = \frac{1}{\widetilde{N}_j} \sum_{t=L_j-1}^{N-1} \left| \widetilde{W}_{j,t}^Y \right|^2 \tag{35b}$$

$$\hat{C}_{XY}(\lambda_j) = \frac{1}{\widetilde{N}_j} \sum_{t=L_j-1}^{N-1} \Re\left(\widetilde{W}_{j,t}^X \, \overline{\widetilde{W}_{j,t}^Y} \right) = \frac{1}{\widetilde{N}_j} \sum_{t=L_j-1}^{N-1} \left(\widetilde{W}_{j,t}^{X0} \, \widetilde{W}_{j,t}^{Y0} + \widetilde{W}_{j,t}^{X1} \, \widetilde{W}_{j,t}^{Y1} \right) \tag{35c}$$

$$\hat{Q}_{XY}(\lambda_j) = -\frac{1}{\widetilde{N}_j} \sum_{t=L_j-1}^{N-1} \Im\left(\widetilde{W}_{j,t}^X \, \overline{\widetilde{W}_{j,t}^Y} \right) = -\frac{1}{\widetilde{N}_j} \sum_{t=L_j-1}^{N-1} \left(\widetilde{W}_{j,t}^{X0} \, \widetilde{W}_{j,t}^{Y1} - \widetilde{W}_{j,t}^{X1} \, \widetilde{W}_{j,t}^{Y0} \right) \tag{35d}$$

To construct confidence intervals for the wavelet coherence we consider the multivariate process:

$$\mathbf{P}_{j,t} = \left[\left| \widetilde{W}_{j,t}^X \right|^2, \left| \widetilde{W}_{j,t}^Y \right|^2, \Re\left(\widetilde{W}_{j,t}^X \, \overline{\widetilde{W}_{j,t}^Y} \right), \Im\left(\widetilde{W}_{j,t}^X \, \overline{\widetilde{W}_{j,t}^Y} \right) \right]^T \tag{36}$$

and the function:

$$g\left([a, b, c, d]^T \right) = \frac{c^2 + d^2}{ab} \tag{37}$$

Then, assuming that $K_{XY}(\lambda_j) > 0$ and applying the delta method, we have (Whitcher, Craigmile 2004):

$$\sqrt{\widetilde{N}_j} \left(\hat{K}_{XY}(\lambda_j) - K_{XY}(\lambda_j) \right) \sim AN(0, R_{abc,j}(0)) \tag{38}$$

where $R_{abc,j}(0) = \nabla g(\mathbf{P}_{j,t})^T \cdot S_{abcd,j}(0) \cdot \nabla g(\mathbf{P}_{j,t})$, $S_{abcd,j}(\cdot)$ is the 4×4 spectral matrix for $\mathbf{P}_{j,t}$ and ∇g is the gradient of g.

Then, an approximate large sample confidence interval for the wavelet coherence is:

$$\hat{K}_{XY}(\lambda_j) \pm \varsigma_{\frac{\alpha}{2}} \left(\frac{\hat{R}_{abcd,j}(0)}{\widetilde{N}_j} \right)^{0.5} \tag{39}$$

where $\varsigma_{\frac{\alpha}{2}}$ is the $(1-\alpha/2)$ quantile of the standard normal distribution.

To construct confidence intervals for the wavelet phase angle we assume that $C_{XY}(\lambda_j) \neq 0$ and take:

$$g\left([a,b,c,d]^T\right) = \tan^{-1}\left(\frac{d}{c}\right) \tag{40}$$

Then, applying the delta method again we arrive at:

$$\sqrt{\widetilde{N}_j}\left(\hat{\theta}_{XY}(\lambda_j) - \theta_{XY}(\lambda_j)\right) \sim AN(0, R_{abc,j}(0)) \tag{41}$$

where the large sample variance, $R_{abc,j}(0)$, is computed as previously utilizing an appropriate vector of partial derivatives.

Approximate large sample confidence intervals are given in a form similar to (39). Multiplying them by a constant will produce approximate confidence intervals for the wavelet time delay. Estimates of the large sample variances of the wavelet spectra can be obtained *via* nonparametric kernel estimation.

4. Empirical examination

In the study of business cycle synchronization between Poland and the euro zone the wavelet analysis of coherence, phase angle and (cross-)correlation was applied.[8] The data used are monthly industrial production indices (Eurostat index NACE Rev. 2 B-D_F) for EA16, EU27, France, Germany, Greece, Ireland, Italy, the Netherlands, Portugal, Spain as well as Poland covering the period 1995.1–2010.06 (186 observations). The series are not seasonally adjusted. In what follows we use logarithmic indices.

Figure 4 presents global estimates of wavelet coherences and time delays. They were computed with the Selesnick's k4l2 filter, except for the first stage, where la12 and its one-sample shifted version was applied. As the HWP filters are longer than conventional wavelet filters, the maximal possible decomposition level in this part of the study is 4, what means that only the shortest business cycles (below 3 years) are examined. They are, however, most important in studies of business cycle synchronization (see Skrzypczyński 2008). Only wavelet coefficients unaffected by circularity are considered. The rectangular kernel estimator was applied in variance estimation with truncation parameters for levels 1–4 equal 1, 2, 4, 8 in the case of the wavelet time delay, while for the wavelet coherence we used 1, 2, 2, 2. These values of truncation parameters were chosen on the base of a small simulation study performed for autoregressive processes connected *via* a regression with delay. In the time delay examination for the case X–Y (see Figure 4) negative values of the delay mean that X leads Y. As we can see, the Polish short cycle leads about 5 months cycles in such countries like France and the Netherlands, while the appropriate leads in the case of Italy and Spain are even longer and equal about 10 months. On the other hand, the Polish cycle lags behind the Greek and Irish cycles and shows a contemporaneous relationship with the German short cycle. The most significant relationship seems to take place in the case of Germany, although even larger coherence values for the last decomposition level were obtained for France

[8] All computations in the empirical part were performed in Matlab. Numerical codes are available upon request. They are built on B. Whitcher's WaveCov package (www2.imperial.ac.uk/~bwhitche/software) and C. Cornish's WMTSA toolkit (www.atmos.washington.edu/~wmtsa).

and the Netherlands. However, the significance of relationships at the last stage of the analysis is hard to confirm as there are only 20 nonboundary wavelet coefficients available at this level.[9] Due to this our results of the wavelet phase angle (wavelet time delay) examination should be treated with caution. Figure 5 presents running wavelet coherences and time delays, which generally point at an increasing correlation at all leads and lags, while at the same time showing a decrease of the lag parameter often below 5 months. Opposite changes take place in the case of Portugal and Spain.

To confirm our results wavelet correlation analysis was also performed. This time the short Daubechies d4 filter was applied, what made it possible to execute a 5 level wavelet decomposition. Besides, we used also the la12 Daubechies filter which has similar squared gain function to the complex k4l2 filter. Firstly, global and running wavelet correlations with d4 were computed (see Figures 6 and 7). Except for some countries like the Netherlands or Ireland the results point at an increasing instantaneous linear dependence between the Polish and euro zone business cycles, especially in the case of the short cycles. They confirm also that the most significant relationship takes place in the case of Germany. Besides, there is an evident jump in the level of instantaneous correlations for scale 8 for the most recent data investigated, what can be attributed to the recent financial crisis (the so-called Great Recession of 2007–2009). This is observed, between others, for correlations with the French and Spanish short cycles. Further, we computed time delay estimates *via* de following formula:

$$\hat{\tau}_j^{WCC} = \arg\max_k \mathrm{Cov}_{\widetilde{W}_j^X \widetilde{W}_j^Y}(k) = \arg\max_k \mathrm{Cov}(\widetilde{W}_{j,t}^X, \widetilde{W}_{j,t+k}^Y) \tag{42}$$

where WCC stands for wavelet cross-correlator.

The estimation was performed with the biased version of (24), which produced slightly better results in short samples in our companion simulation analysis than the unbiased wavelet cross-covariance estimator (see Bruzda 2011). Only nonboundary wavelet coefficients were used. The results are given in Table 1, where for comparison purposes we included also estimates of the wavelet time delay in the form:

$$\hat{\tau}_j^{WPA} = -\frac{\hat{\theta}_{XY}(\lambda_j)}{2\pi f_{j0}} \tag{43}$$

where WPA denotes an estimation *via* the wavelet phase angle (see also Figure 4).

Although the results obtained with the three wavelets and the two estimation methods differ substantially, they are usually in agreement as for the sign of the delay parameter, even if different decomposition levels are considered. In particular, the wavelet cross-covariance based delay estimates point at a lead time of length 1–2 months between the Polish and the euro zone (EA16) short business cycles, while the WPA-based method indicates a lead of about half a year. In interpreting the results one should take into consideration several sources of estimation errors and how they influence the estimates. Firstly, different wavelets estimate cyclical components differently. In particular, the d4 wavelet filter is much more imprecise in isolating frequency

[9] When testing for significance of the wavelet coherence for the last scale with the block bootstrap method, we were not able to reject the nonsignificance of the wavelet coherence at the nominal level of 10% for each of the series examined. We resampled blocks of length 8 from differenced observations and then reintegrated the series. The overlapping block bootstrap method was applied.

components of time series as compared to la12 (see the squared gain functions of the two wavelet filters in Figure 2), while at the same time it is 3 times shorter than la12, what results in a much larger number of wavelet coefficients unaffected by the extrapolation method at the ends of the samples. Secondly, simulation experiments show that in the case of low signal-to-noise ratios the WPA delay estimator is biased in small samples towards 0, while the WCC method, although asymptotically optimal, is characterised by a much larger small sample variance, what leads also to larger MSE (see simulation analysis in our companion paper). Generally, when a more precise estimation of frequency components is needed, the wavelet time delay (43) seems to be a better choice for time delay estimation in small and medium size samples.

It is worth mentioning that, to a large extent, our results are in line with the majority of other studies on business and growth cycle synchronization that include Poland and the euro area, especially those in Hughes Hallett, Richter (2007), Woźniak, Paczyński (2007), Skrzypczyński (2008), Adamowicz et al. (2009) and Konopczak (2009). In particular, Hughes Hallett, Richter (2007) and Woźniak, Paczyński (2007), using the short-term Fourier transform, document a high coherence of growth cycles for Poland and the euro zone in the post-transition period for fluctuations of length coinciding with the typical horizon of monetary policy. Skrzypczyński (2008) with the help of spectral and correlation analysis finds that the Polish short and long deviation cycles lead the corresponding euro zone cycles and that the degree of synchronization is higher for the short cycle (up to 3 years) that for the long one (6–7 years in length). Besides, he documents that other countries in our region seem to show a slightly faster convergence to the European cycle than the Polish economy. In turn, Adamowicz et al. (2009) in a voluminous investigation find a moderately high level of synchronization with the euro zone as a whole, while the highest level of dependence has been affirmed in the case of Germany. In addition, they notice that the Polish cycle usually leads the cycling components of the euro zone member countries. Besides, they provide a tabular listing of empirical studies on business cycle synchronization for Poland and the EU countries together with their major finding (p. 91, Table 5.1). Finally, Konopczak (2009) points at a generally high level of business cycle synchronization between Poland and the euro area measured with concordance indices and correlations of recession probabilities, especially in comparison with other CEE countries investigated (the Czech Republic, Estonia, Latvia, Lithuania and Slovakia), although the levels of demand and supply shocks correlations are rather low.

In the second part of the empirical study we concentrate exclusively on the euro zone countries and analyse the same production indices in the longer period from January 1991 till June 2010, what gives a total of 234 observations, except for Greece, for which data starting from 1995 were only available. The longer data set made it possible to include the sixth decomposition level into our examination. In this part of the study we used wavelet analysis of variance and correlation as well as wavelet coherence and phase angle examination. Firstly, we computed the running wavelet variances for scales 8, 16 and 32 corresponding to decomposition levels 4–6 (see Figure 8). The d4 Daubechies wavelet filter was applied. Looking at the wavelet variances it is possible to find certain general patterns of volatility changes in business cycle components of the series under scrutiny, at least for the short- and medium-term business cycles: for the majority of the economies in our sample the short business cycle fluctuations show fairly stable level of volatility, except for the very beginning of the series as well as the most recent period of the Great Recession of 2007–2009, while at the same time the fluctuations up to 5 years in length are decreasing in their amplitudes. Among exceptions there are Greece and the Netherlands, for which we observe an increasing wavelet variance for both the short- and medium-term fluctuations. For certain countries in the sample the jump in volatility of business cycles due to the Great Recession is substantial (for example, Germany as well as the euro area as a whole), although there are also countries with business cycles that seem no to be affected by the crises (Portugal and Ireland).

The business cycle convergence within the euro zone was examined with the help of running wavelet correlations as well as running wavelet coherences and time delays (see Figures 9–10). For each country in the sample the changing levels of business cycle synchronization with the euro zone (EA16) were examined. The results of wavelet correlation analysis point at a quite stable synchronization in the case of the short cycles, with a slight increasing tendency in the most recent period that can be attributed to the contagion effects and the global financial crisis, while at the same time we may observe a decreasing level of correlations for cycles above 5 years in length. The lack of a more systematic changes in the instantaneous dependencies makes the endogeneity hypothesis of the OCA criteria hard to verify. To this end longer data series will be necessary. The local wavelet analysis of coherence and time delay, performed for the shortest cycles only, points additionally at approximately constant phase relationships between each country cycle and the European business cycle, except for Ireland and Portugal, in the case of which a lagged relationship turned to a leading one, i.e. for the most recent data the euro zone cycle is lagging behind these countries cycles. Also the strength of linear dependences at all leads and lags measured with the wavelet coherence do not changes much.

5. Conclusions

The bivariate continuous discrete wavelet analysis provides a summary of evolutionary cross--spectral characteristics of the processes under scrutiny with good localization properties, high computational efficiency and without an excessive redundancy of information that takes place, e.g., in the continuous wavelet analysis. These features make the approach particularly attractive in processing economic time series which are often subject to structural breaks, local trends and changing cyclical characteristics. In the paper we made use of these properties in order to describe changing patterns of business cycle synchronization between Poland and 8 euro zone member countries. Besides, we addressed also questions concerning the endogeneity hypothesis of the optimum currency area criteria as well as the recent changes in business cycle variability. In our study wavelet coherence and phase angle examination in the global and local (short-term) versions was applied. The study was supplemented with wavelet analysis of variance and wavelet correlation and cross-correlation examination. Our empirical results point at an increasing synchronization of the Polish business cycle with the euro zone cycles as well as a fairly stable level of business cycle synchronization among the euro zone countries themselves. However, to obtain more firm results much longer data sets are needed, as the wavelet analysis of bivariate spectra is rather data demanding.

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Appendix

Table 1

Time delay estimates for levels 4 and 5

	la12/k4l2				*d*4
Country Y	$\hat{\tau}_4^{WCC}$	$\max_k \hat{\rho}_k(\lambda_4)$	$\hat{\rho}(\lambda_4)$	$\hat{\tau}_4^{WPA}$	$\hat{\tau}_5^{WCC}$
EA16	-2	0.968	0.647	-5.734	-4
EU27	-1	0.970	0.787	-5.029	-3
Germany	-3	0.737	0.737	0.199	-2
France	-12	0.341	0.100	-6.316	-7
Italy	-12	0.241	-0.163	-9.306	-12
Spain	-10	0.419	0.371	-10.369	-4
The Netherlands	0	0.610	0.570	-5.667	0
Portugal	-1	0.246	0.141	-8.618	-18
Greece	6	0.176	0.449	2.276	-1
Ireland	0	0.902	0.902	1.778	0

Note: The wavelet cross-covariance based estimator was computed over the ranges: -12:12 (level 4) and -24:24 (level 5). Numbers of nonboundary wavelet coefficients are the following: 20 (level 4, la12 and k4l2 wavelets), 92 (level 5, d4 wavelet).

Figure 1

Scaling and wavelet functions for two Daubechies filters: d4 and la12

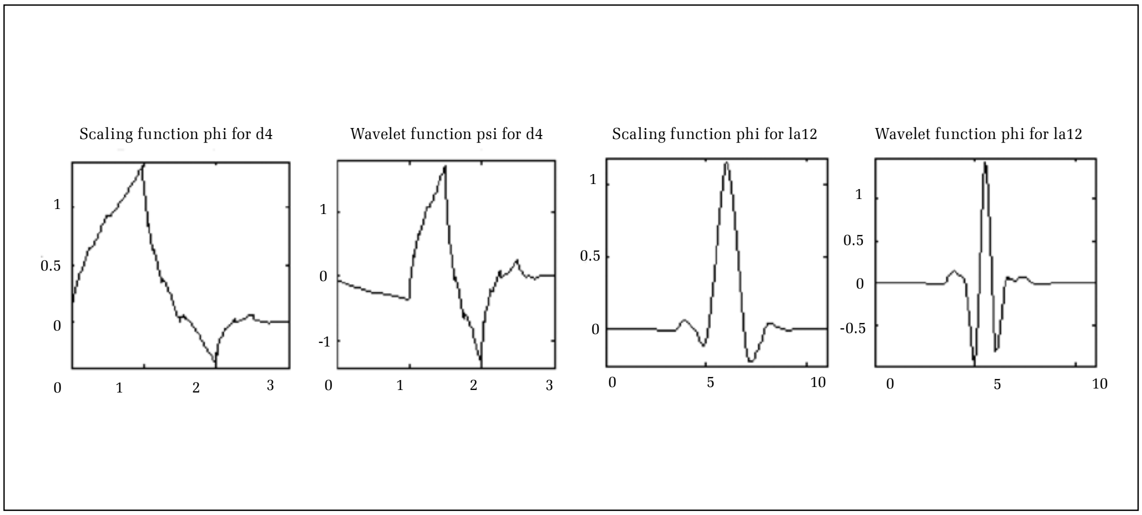


Figure 2

Squared gain functions for two Daubechies filters of length 4 and 12; the MODWT case

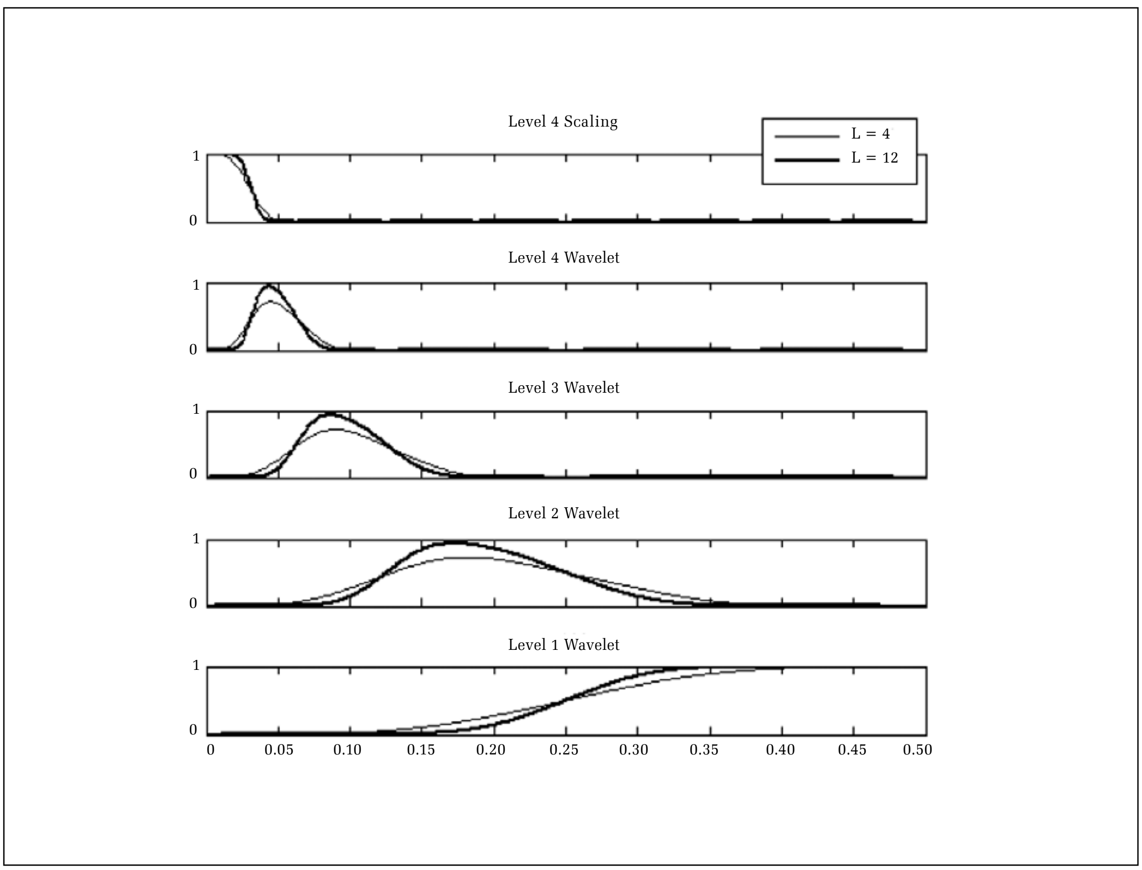
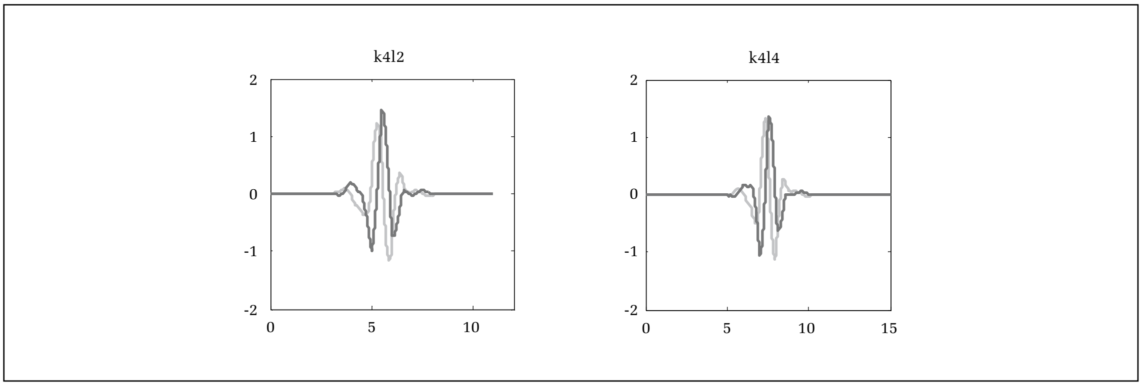


Figure 3

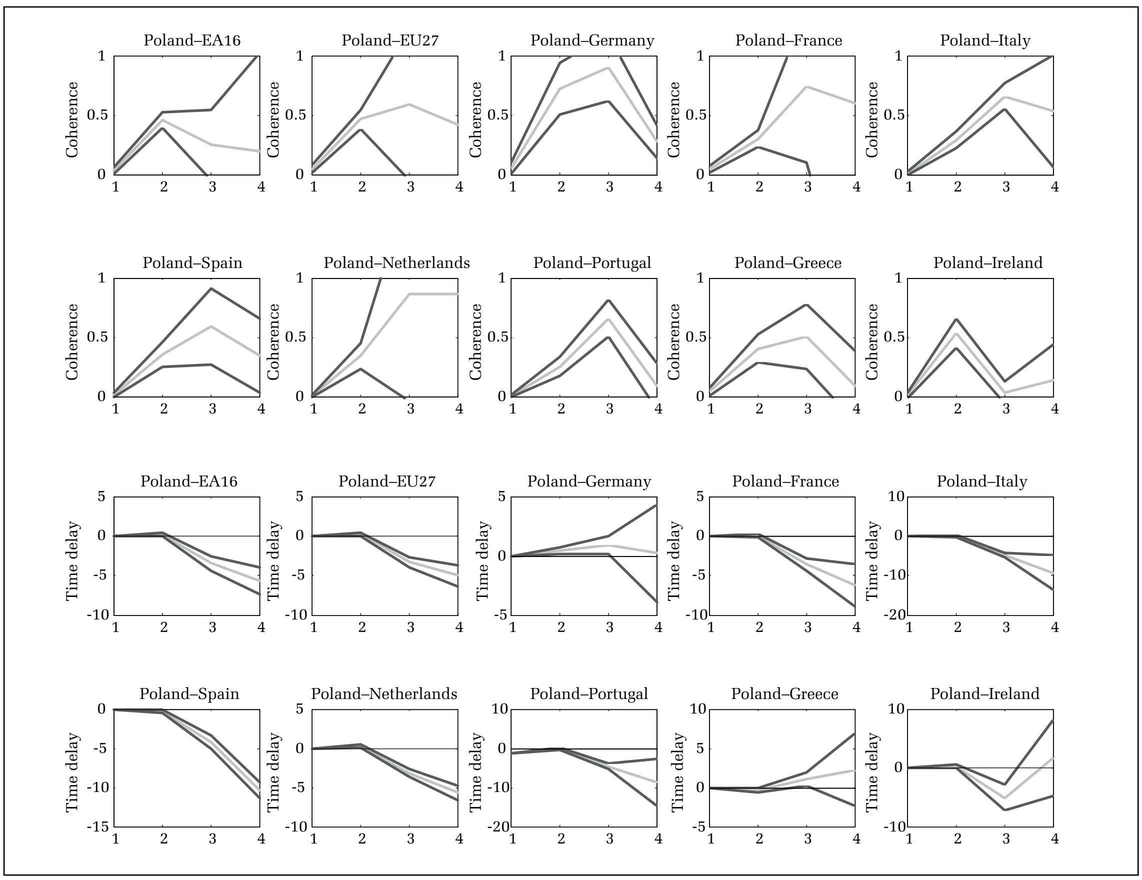
Example Hilbert wavelet pairs



Note: see the description in the text.

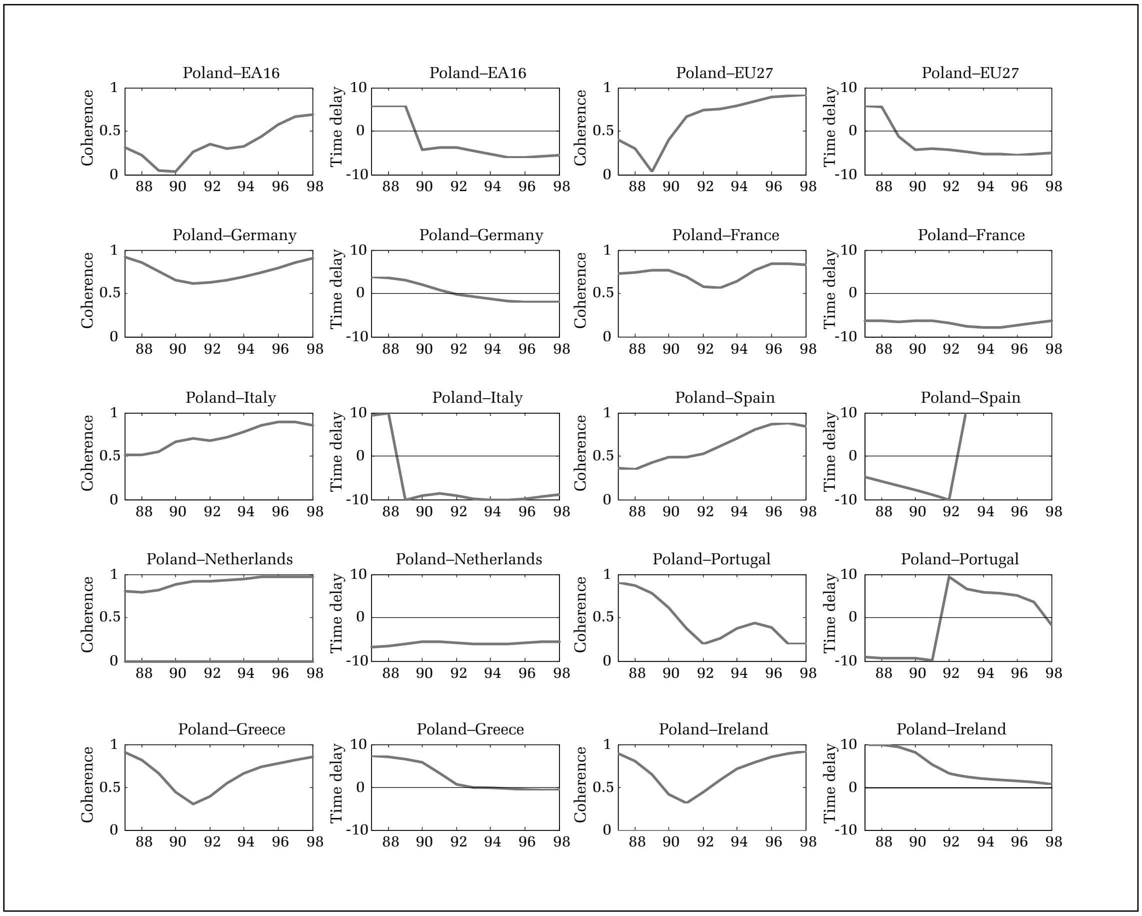
Figure 4

Wavelet coherences and wavelet time delays



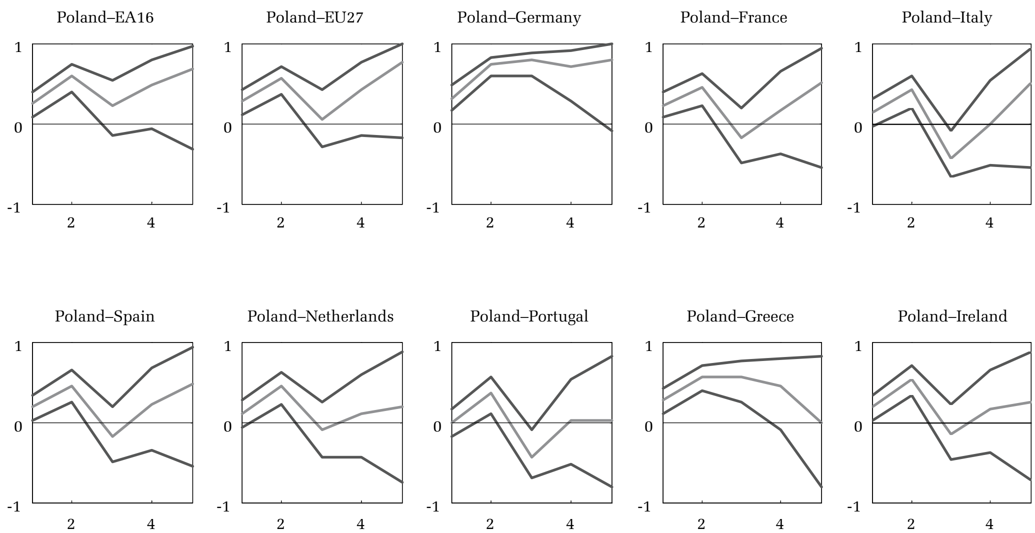
Note: see formulas (33) and (34) for decomposition levels 1–4 corresponding to oscillations with period lengths 2–4 (up to a quarter), 4–8 (about half a year), 8–16 (about 1 year) and 16–32 (below 3 years) together with large sample 90% confidence intervals; numbers on horizontal axes are the decomposition levels *j*.

Figure 5
Running wavelet coherences and wavelet time delays



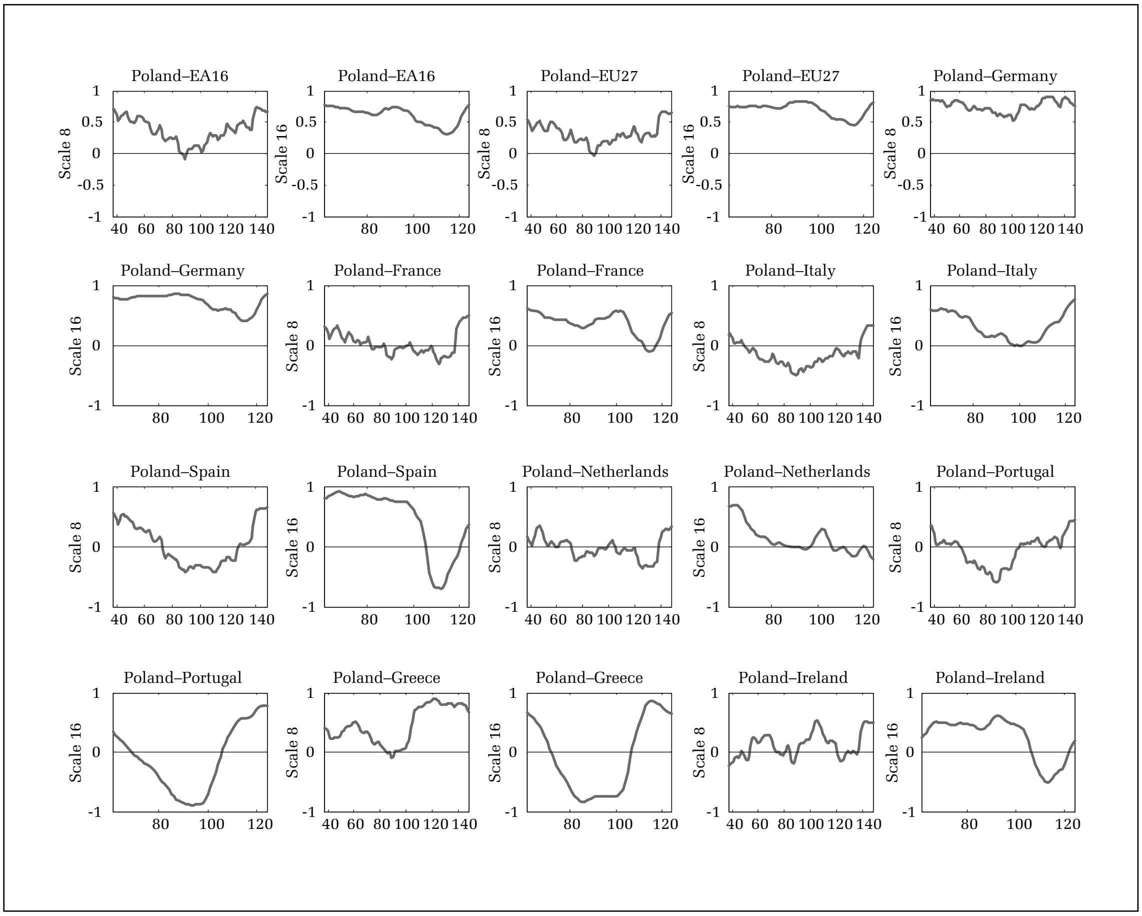
Note: see formulas (33) and (34) for level 4 in windows consisting of 10 nonboundary wavelet coefficients after circular shifting in order to align them to the original data; numbers on horizontal axes are the mid-points of the subsamples of circularly shifted wavelet coefficients.

Figure 6
Wavelet correlations



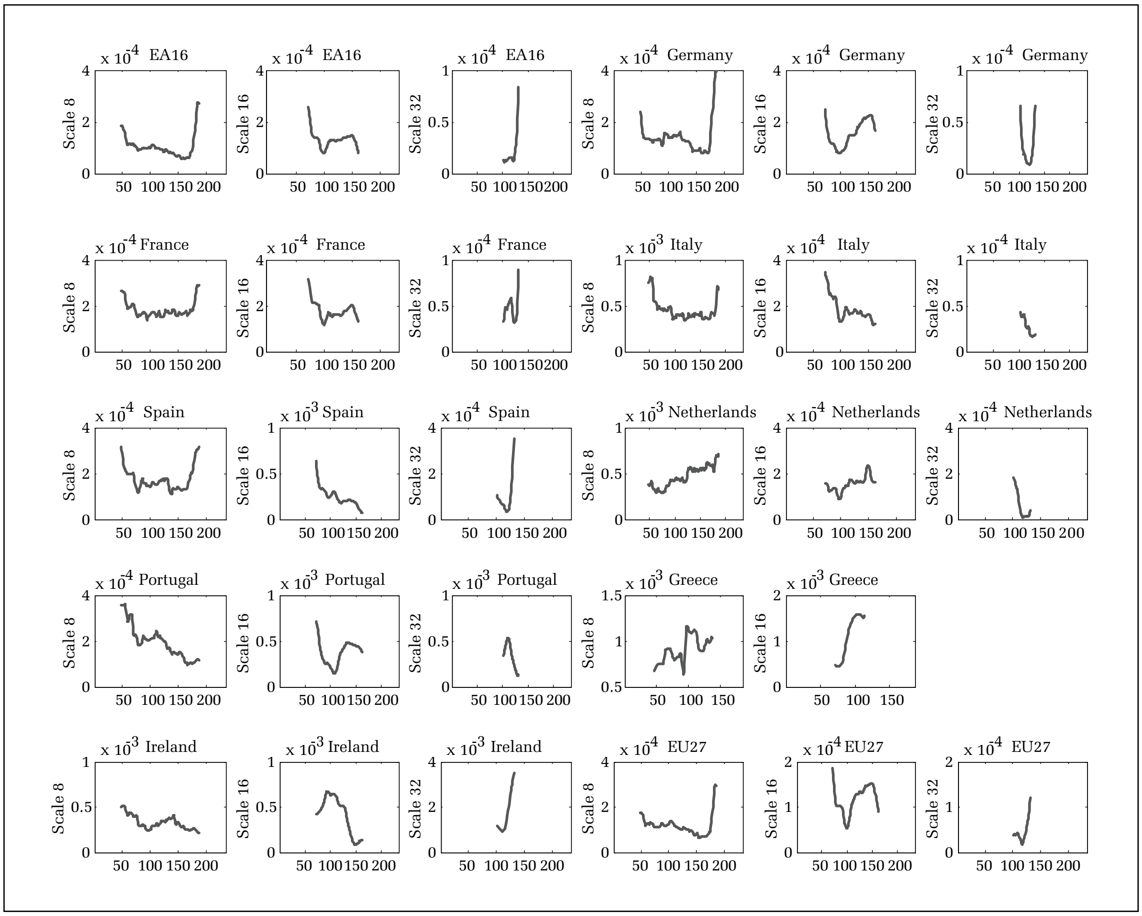
Note: see formula (13) for decomposition levels 1–5 corresponding to oscillations with period lengths 2–4 (up to a quarter), 4–8 (about half a year), 8–16 (about 1 year), 16–32 (below 3 years) and 32–64 (up to 5 years) together with 90% confidence intervals computed with the inverse hyperbolic tangent transformation – see formula (23); numbers on horizontal axes are the decomposition levels *j*.

Figure 7
Running wavelet correlations



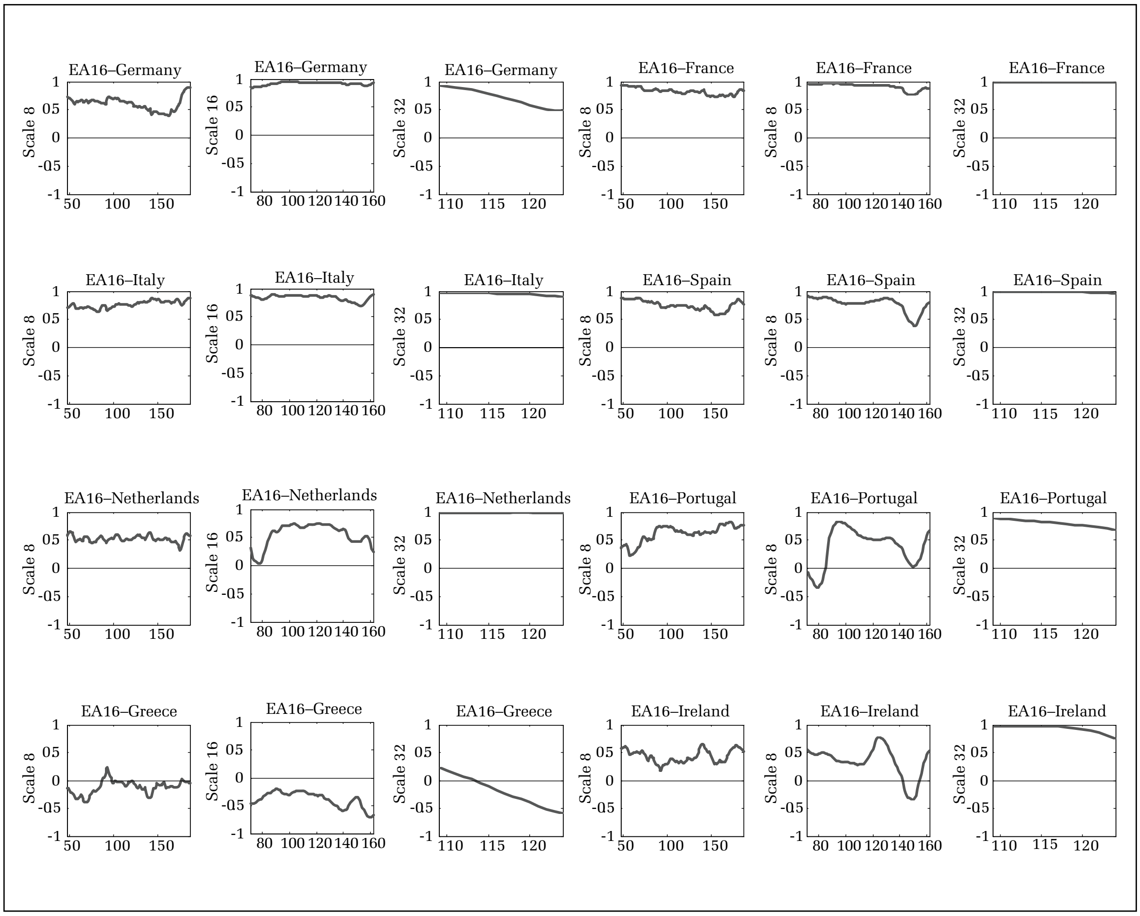
Note: see formula (13) for scales 8 and 16 corresponding to oscillations with period lengths 16–32 (below 3 years) and 32–64 (up to 5 years); computation in windows of 30 nonboundary wavelet coefficients after aligning them to the original data; numbers on horizontal axes are the mid-points of the subsamples of circularly shifted wavelet coefficients.

Figure 8
Running wavelet variance



Note: see formula (9) for scales 8, 16 and 32 corresponding to oscillations with period lengths 16–32 (below 3 years), 32–64 (up to 5 years) and 64–128 (up to 10 years); results obtained with d4 Daubechies wavelet filter of length 4 and windows of 50 wavelet coefficients for levels 4–5 (15 coefficients for level 6) unaffected by circularity after aligning them to the observations in the sample; numbers on horizontal axes are the mid-points of the subsamples of circularly shifted wavelet coefficients.

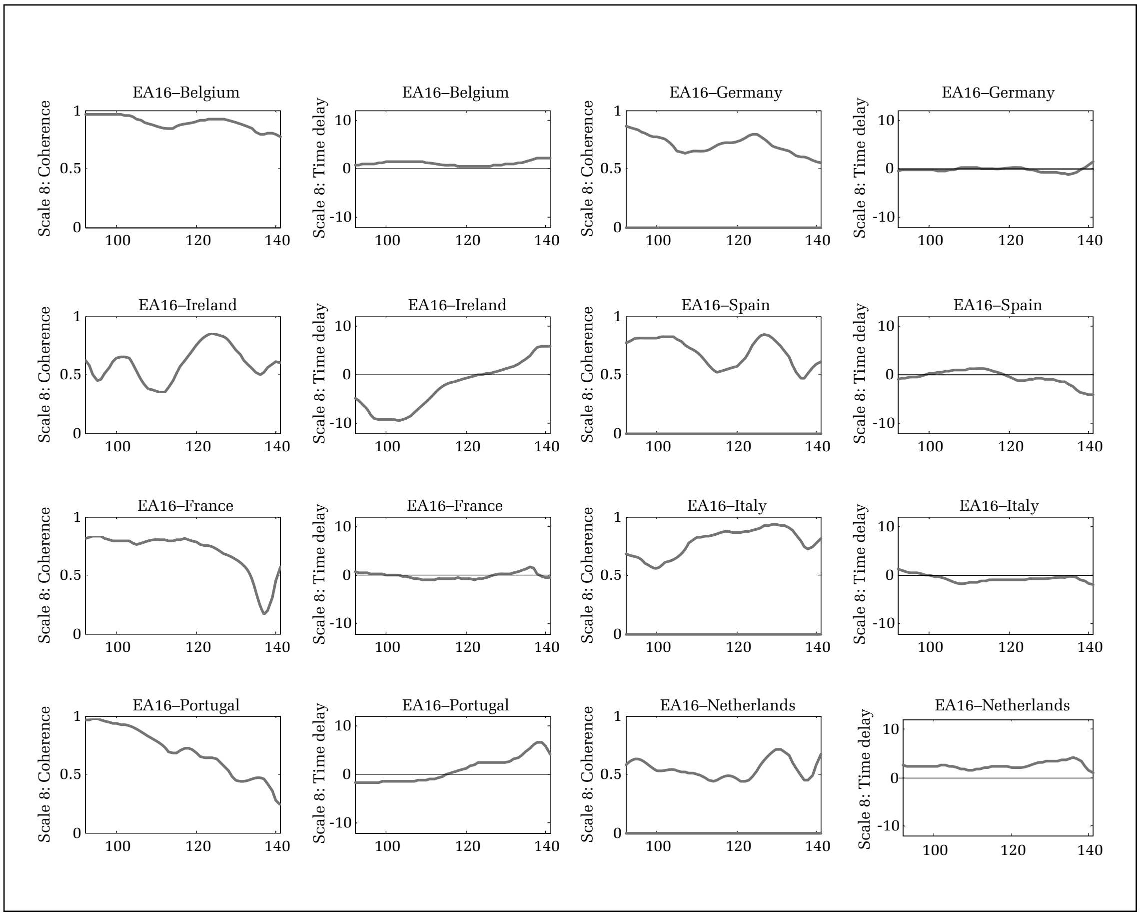
Figure 9
Running wavelet correlations



Note: see formula (13) for scales 8, 16 and 32 corresponding to oscillations with period lengths 16–32 (below 3 years), 32–64 (up to 5 years) and 64–128 (up to 10 years); results obtained with d4 Daubechies wavelet filter of length 4 and windows of 50 wavelet coefficients unaffected by circularity after aligning them to the observations in the sample; numbers on horizontal axes are the mid-points of the subsamples of circularly shifted wavelet coefficients.

Figure 10

Running wavelet coherences and wavelet time delays



Note: see formulas (33) and (34) at level 4 in windows consisting of 10 nonboundary wavelet coefficients after circular shifting in order to align them to the original data; numbers on the horizontal axis are the mid-points of the subsamples; numbers on horizontal axes are the mid-points of the subsamples of circularly shifted wavelet coefficients.